MAT 502 - Review Sheet

Disclaimer: The problems below are meant to illustrate some of the main concepts of MAT 502. They should not necessarily be considered as examples of what is going to be on the comprehensive exam.

1. Prove that $\sqrt{21}$ is irrational.

2. Let P be a regular polygon with n sides inscribed in a circle of radius 1, and let Q be a regular polygon with $2n$ sides inscribed in a circle of radius 1. If the length of each side of P is s find a formula for the length of each side of Q in terms of s. Apply the formula again and again to determine the perimeter of a regular $24-gon$ inscribed in a circle of radius 1. What value for π does this give?

3. Do the following in neutral geometry:
   (a) Prove that if one of the sides of a triangle is produced, then the exterior angle is greater than either of the opposite interior angles.
   (b) Prove that two angles of a triangle taken together are less than two right angles.

4. State five substantially different statements each of which is equivalent to Euclid's fifth postulate and prove that two of these are indeed equivalent to Euclid's fifth.

5. How many regular polyhedra are there? Prove your answer.

6. Prove that there are infinitely many primes of the form $6k+5$, where k is an integer.

7. What are Mersenne primes? State and prove a theorem involving Mersenne primes.

8. Do the following:
   (a) Using the Euclidean Algorithm, find $GCD(72, 48, 60)$.
   (b) Write $GCD(72, 48, 60)$ as a linear combination of 72,48, and 60.

9. 8-GB USB flash drives sell for \$12, while 16-GB USB flash drives sell for \$25. You spent \$331 altogether. How many drives of each type did you buy? Set us a Diophantine equations and solve it using the Euclidean algorithm.

10. Find all solutions of the congruence equation $32x \equiv 12 \, mod \, 70$.

11. Use congruence to do the following:
    (a) Prove that 41 divides $2^{20}-1$.
    (b) What are the last two digits of 3^{1001}?
    (c) What is the least nonnegative residue mod 79 of 3^{100}?
    (d) What is the remainder when 2^{10000} is divided by 31?

12. Suppose that you intercept the number 9 being transmitted to someone who uses the RSA algorithm and has published, "Anyone who wants to convey to me the number x should transmit the least nonnegative residue of $x^{11} \, mod \, 65$." What number was being conveyed?

13. Do the following:
    (a) Compute the similarity and box dimensions for your favorite fractal.
    (b) Describe how to make a fractal curve with similarity dimension 3/2. Draw the first 2 steps in the formation of this curve.
    (c) Let $F_c(z) = z^2 + c$. Define the Julia set of F_c and the Mandelbrot set of F_c.