1. Given any n odd numbers $a_1, a_2, \ldots, a_n$, where $n > 1$ is an integer, the product $a_1 \cdot a_2 \cdot \ldots \cdot a_n$ is odd. (Use induction.)

2. Prove that the number $\sqrt{10}$ is irrational.

3. Prove that the number $\sqrt{12}$ is irrational.

4. Let R be a relation on $\mathbb{Z}$ defined by aRb if and only if $a^2 + b^2$ is even. Then R is an equivalence relation and $\{0, 1\}$ is a complete set of representatives.

5. Given any two nonempty sets A and B and any function $f\colon A \to B$, the relation R on A defined by xRy if and only if $f^{-1}(f(x)) = f^{-1}(f(y))$ is an equivalence relation on $\mathbb{R}$.

6. The function f defined by $f(x) = \sqrt{x^2 - 2}$ is neither injective nor surjective from the natural domain of f to the reals.

7. Let a and b be integers and let n be any positive integer greater than one.
   a. Prove that if a is a prime number, then $a|b$ if and only if $a|b^n$.
   b. Provide an example where a is not prime and the if and only if fails to be true.

8. Let $f\colon X \to Y$ be a function and let $A, B \subseteq X$.
   a. Prove that $f(A \cap B) \subseteq f(A) \cap f(B)$.
   b. Provide an example where $f(A \cap B) \subset f(A) \cap f(B)$.
   c. Prove that if f is also injective, then $f(A \cap B) = f(A) \cap f(B)$.

9. Prove (using any relevant results) that $\{n^2 \mid n \in \mathbb{N}\}$ is denumerable.

10. Prove (using any relevant results) that $\{x^2 \mid x \in \mathbb{R}\}$ is uncountable.

11. Prove that $\mathbb{Q}$ is denumerable without referring to any uncountable sets.

12. Prove that $(0, 1)$ is uncountable without referring to any other uncountable sets.

13. Let R be the relation on $\mathbb{R}$ defined by aRb if and only if $a - b \in \mathbb{Z}$.
    a. Prove that R is an equivalence relation.
    b. Find a complete set of representatives of $\mathbb{R}/R$.
    c. Describe $[\pi]$ using set notation, or graph $[\pi]$ as a subset of $\mathbb{R}$.

14. Let $R = \{(0, \{0\}), (1, \{1\})\}$ and $S = \{(\{0\}, 0), (\{1\}, 1), (\{0, 1\}, 0), (\{0, 1\}, 1)\}$.
    a. Find $S^{-1}(0)$.
    b. Find $S \circ R$.
    c. Prove or disprove: $S \circ R$ is a partial order.
    d. Prove or disprove: $S \circ R$ is an equivalence relation.

15. Let $f(x) = e^{2x}$ and let $g(x) = \ln(x)/2$. Using properties of exponential and logarithmic functions, show that $g \circ f = i_{\mathbb{R}}$. Does this mean that f and g are bijections on $\mathbb{R}$ and that $g = f^{-1}$? If so, why? If not, what can be said about f and g in terms of the properties: injective, surjective, bijective, inverses.

16. Prove or disprove that $[(P \Rightarrow Q) \wedge \sim P] \Rightarrow Q$ is a valid argument.

17. Prove or disprove the following statement: *If p and q are prime numbers, then $\sqrt{pq} \in \mathbb{Q}$ if and only if $p = q$.*

18. Prove or disprove the following statement: *There exists an equivalence relation R on a nonempty set A for which there are elements $x, y, z \in A$ such that xRy and yRz and $(z, x) \notin R$.*

19. Prove or disprove the following statement: *The relation R on $\mathbb{R}$ defined by $a\ R\ b$ if and only if $a + b \in \mathbf{Z}$ is an equivalence relation.*

20. Given the usual definition of an ordered pair $(x, y) = \{\{x\}, \{x, y\}\}$, we define an ordered triple
$$(x, y, z) = (x, y) \cup \{\{x, y, z\}\}.$$
Explain why each of the following is either true or false.
    a. $(1, 2) \in (1, 2, 3)$
    b. $(1, 2) \subset (1, 2, 3)$

21. Consider the relation $R = \{(a, b), (b, c), (c, b)\}$ on the set $A = \{a, b, c\}$.
    a. Find the smallest transitive relation that contains R.
    b. Disprove the proposition: *There exists a partial order that contains R.*

22. Prove the following. Let x be any irrational real number and let r be any rational number. Then rx is irrational.

23. Prove the proposition: *Let A be a set and let $\{A_i\}_{i \in \mathbb{N}}$ be a collection of sets such that for each i, $A_i \subseteq U$ for some set U. Then*
$$\left(\bigcup_{i \in \mathbb{N}} A_i\right)^c = \bigcap_{i \in \mathbb{N}} A_i^c.$$

24. Prove that for any integer $n \geq 2$, the product of n real numbers is 0 if and only if at least one of the numbers is 0.